Problem 1: Complex numbers

1. Determine real and imaginary parts

   a) $\dfrac{1-j}{1+j} = -j$ b) $\dfrac{2}{1-3j} = \dfrac{1}{5}(1+3j)$ c) $\left(1+j\sqrt{3}\right)^3 = -8$

2. Determine in polar coordinates

   a) $|\underline{z}| = 3$ and $\varphi = \dfrac{\pi}{2}$ b) $|\underline{z}| = 2$ and $\varphi = -\dfrac{\pi}{2}$

   c) $|\underline{z}| = \sqrt{2}$ and $\varphi = \dfrac{\pi}{4}$ d) $|\underline{z}| = \sqrt{2}$ and $\varphi = -\dfrac{3\pi}{4}$

   e) $|\underline{z}| = \sqrt{29}$ and $\varphi = \arctan\left(\dfrac{5}{2}\right)$ f) $|\underline{z}| = \sqrt{29}$ and $\varphi = -\arctan\left(\dfrac{5}{2}\right)$

   g) $|\underline{z}| = \sqrt{29}$ and $\varphi = \pi - \arctan\left(\dfrac{5}{2}\right)$ h) $|\underline{z}| = \sqrt{29}$ and $\varphi = \arctan\left(\dfrac{5}{2}\right) - \pi$

   i) $|\underline{z}| = |b|$ and $\varphi = \dfrac{\pi}{2}\,\mathrm{sgn}\,(b)$

   k) $|\underline{z}| = \sqrt{a^2+b^2}$ and $\varphi = \begin{cases} \arctan\left(\dfrac{b}{a}\right) & \text{, if } a > 0 \\ \arctan\left(\dfrac{b}{a}\right) + \pi & \text{, if } a < 0,\ b \geq 0 \\ \arctan\left(\dfrac{b}{a}\right) - \pi & \text{, if } a < 0,\ b < 0 \end{cases}$

3. Complex conjugated

   Let $\underline{z}_1 = \dfrac{1}{2}\left(1-\sqrt{3}j\right)$ and $\underline{z}_2 = 2e^{j\pi/6}$

   a) Compute $\underline{z}_1 \cdot \underline{z}_2 = \sqrt{3} - j$

   b) Compute $\underline{z}_1 \cdot \overline{\underline{z}}_2 = -2j$

   c) Determine magnitude and phase of $\overline{\underline{z}}_1 \cdot \underline{z}_2 = 2j$

   d) Determine real and imaginary part of $\overline{(1+\underline{z}_1)\cdot(1-\overline{\underline{z}}_2)} = \dfrac{3}{2} - \sqrt{3} + j\dfrac{\sqrt{3}}{2} - j3$

4. Calculate n-th root of a complex number

a) $\sqrt[3]{j} = \begin{cases} \frac{\sqrt{3}}{2} + j\frac{1}{2} \\ -\frac{\sqrt{3}}{2} + j\frac{1}{2} \\ -j \end{cases}$ b) $\sqrt[4]{-1} = \begin{cases} \pm\frac{\sqrt{2}}{2}(1+j) \\ \pm\frac{\sqrt{2}}{2}(1-j) \end{cases}$

c) $\sqrt[6]{-8} = \begin{cases} \pm\frac{\sqrt{2}}{2}\left(\sqrt{3}+j\right) \\ \pm\frac{\sqrt{2}}{2}\left(\sqrt{3}-j\right) \\ \pm\sqrt{2}j \end{cases}$ d) $\sqrt[8]{1} = \begin{cases} \pm\frac{\sqrt{2}}{2}(1+j) \\ \pm\frac{\sqrt{2}}{2}(1-j) \\ \pm 1 \\ \pm j \end{cases}$

e) $\sqrt{1-j} = \pm\frac{\sqrt{2}}{2}\left(\sqrt{\sqrt{2}+1} - j\sqrt{\sqrt{2}-1}\right)$

f) $\sqrt{3+4j} = \pm(2+j)$

g) $\sqrt[3]{-2+2j} = \sqrt{2}\left(\cos\left(\frac{2\pi k + \frac{3\pi}{4}}{3}\right) + j\sin\left(\frac{2\pi k + \frac{3\pi}{4}}{3}\right)\right)$, k = 0, 1, 2

5. Find real and imaginary parts of the following functions

a) $\cos(2+j) = \cos 2\cosh 1 - j\sin 2\sinh 1$

b) $\sin(2j) = j\sinh 2$

c) $\tanh(2-j) = \frac{\sinh 4 - j\sin 2}{2\left(\cos^2 2 + \sinh^2 1\right)}$

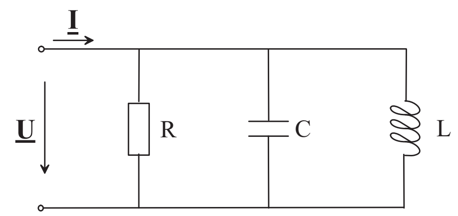
d) $\cot\left(\frac{\pi}{4} - j\cdot\ln 2\right) = \frac{8+j15}{17}$

e) $\coth(2+j) = \frac{\sinh 4 - j\sin 2}{\cosh 4 - \cos 2}$

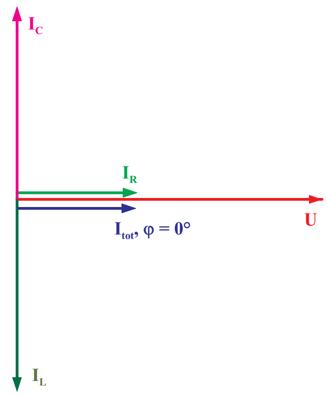
Problem 2: Phasor diagram

Find the phasor diagrams of the given networks and determine the phase shift between the current I and the voltage U.

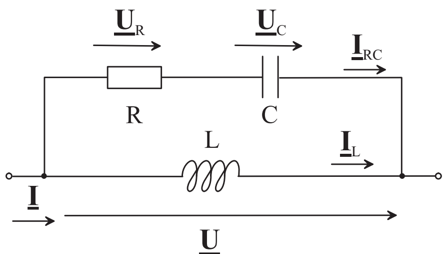
a)



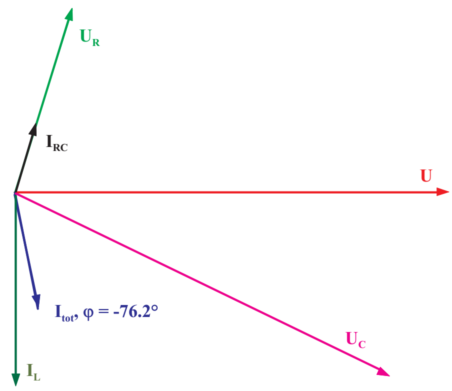
$$\text{with } \begin{aligned} R &= 2.5\ \Omega \\ C &= 7.9\ \text{pF at f} = 12.7\ \text{GHz} \\ L &= 20\ \text{pH} \end{aligned}$$



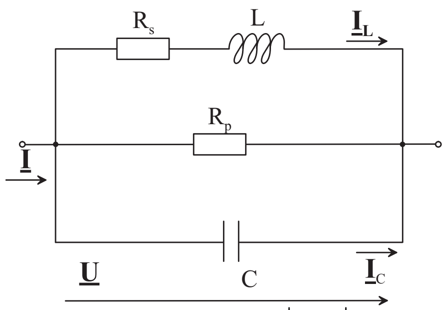
b)



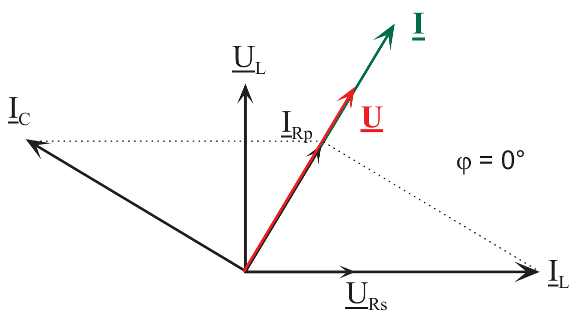
$$\text{with } \begin{aligned} R &= 25\ \Omega \\ C &= 2.5\ \text{pF at f} = 1\ \text{GHz} \\ L &= 4.5\ \text{nH} \end{aligned}$$



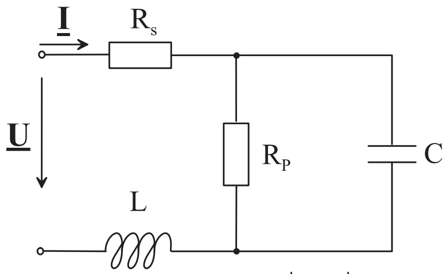
c)

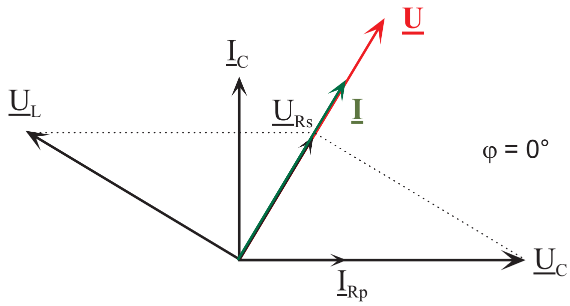


$$\text{with } \begin{aligned} |\underline{I}_C| &= \sqrt{3} \cdot \left|\underline{I}_{R_p}\right| \\ |\underline{I}_L| &= 2 \cdot \left|\underline{I}_{R_p}\right| \\ |\underline{U}_L| &= \sqrt{3} \cdot \left|\underline{U}_{R_S}\right| \end{aligned}$$



d)

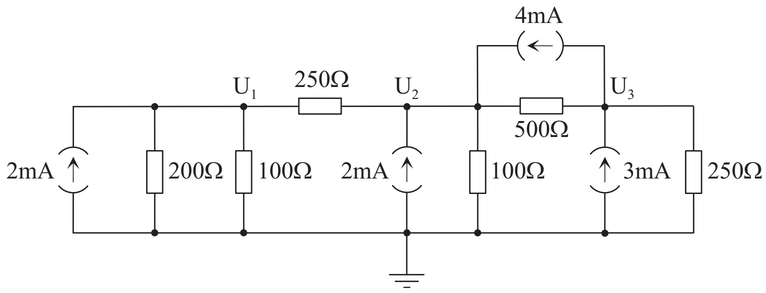




$$\text{with} \quad \begin{aligned} |\underline{I}_C| &= \sqrt{3} \cdot \left|\underline{I}_{R_p}\right| \\ |\underline{U}_C| &= 2 \cdot \left|\underline{U}_{R_S}\right| \\ |\underline{U}_L| &= \sqrt{3} \cdot \left|\underline{U}_{R_S}\right| \end{aligned}$$

Problem 3: Nodal analysis

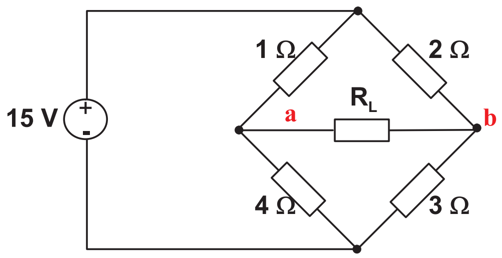
Find the solution of the network given below using nodal analysis. Determine all voltages (U_1, U_2, U_3).

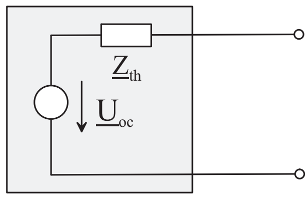


$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0.1937 \\ 0.4201 \\ -0.0266 \end{bmatrix}$$

Problem 4: Network theorem

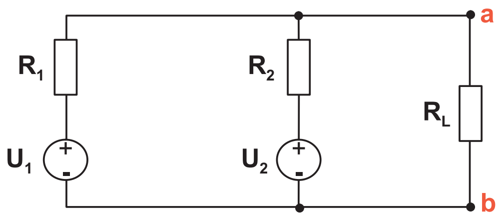
1. Draw a Thévenin equivalent circuit for the circuit in the figure below and determine the value of resistance R_L which will allow a current of 1 A to flow between **a** and **b**.

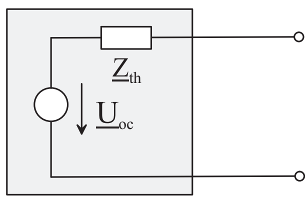




$U_{oc} = 3$ V, $\underline{Z}_{th} = 2\Omega$, $R_L = 1\Omega$

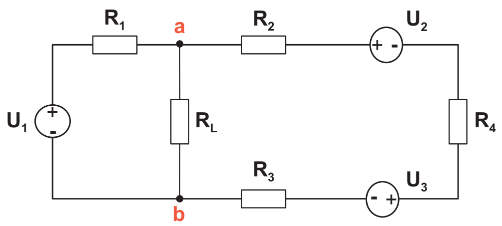
2. Find the Thévenin equivalent circuit for the network to the left of terminals **a** and **b**, if $U_1 = 10$ V, $U_2 = 15$ V, $R_1 = 4\ \Omega$ and $R_2 = 6\ \Omega$.

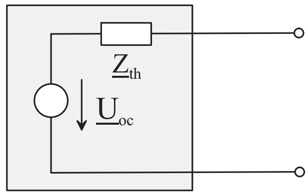




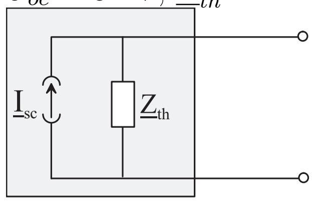
$U_{oc} = 12$ V, $\underline{Z}_{th} = 2.4\Omega$

3. Find Thévenin and Norton equivalent circuits for the network between terminals **a** and **b**, if $U_1 = 100$ V, $U_2 = 50$ V, $U_3 = 10$ V, $R_1 = 4\ \Omega$, $R_2 = 2.2\ \Omega$, $R_3 = 2.3\ \Omega$ and $R_4 = 1.5\ \Omega$.



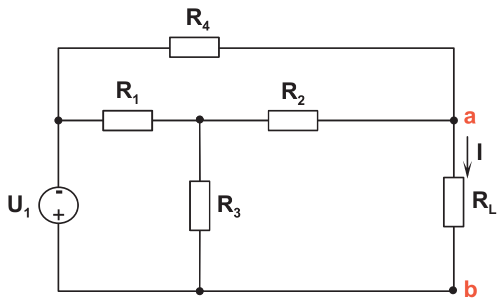


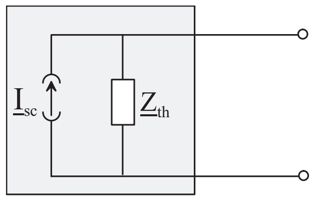
$U_{oc} = 84$ V, $\underline{Z}_{th} = 2.4\Omega$



$I_{sc} = 35$ A, $\underline{Z}_{th} = 2.4\Omega$

4. Find a Norton equivalent circuit for the network between terminals **a** and **b**, if $U_1 = 2$ V, $R_1 = 3\ \Omega$, $R_2 = 1\ \Omega$, $R_3 = 6\ \Omega$ and $R_4 = 6\ \Omega$. Use this result to find current I.





$$I_{sc} = -\frac{1}{9}\text{ A},\ \underline{Z}_{th} = 2\Omega$$

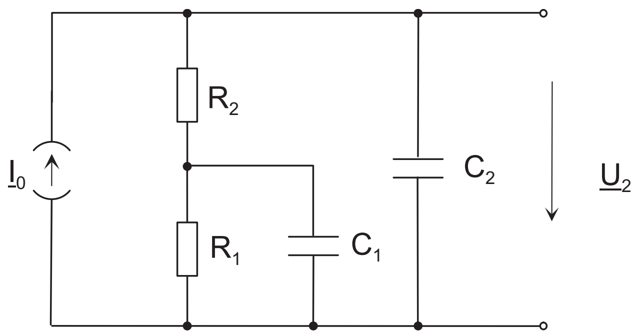
$$I = -\frac{2}{9}\text{ A}$$

Problem 5: Frequency response

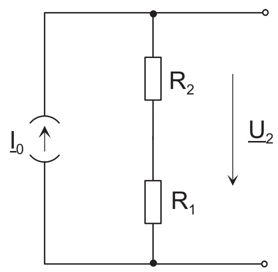
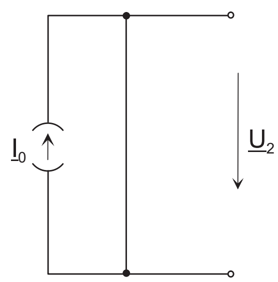
In this problem a transfer impedance determined as

$$Z_T(\omega) = \frac{\underline{U}_2}{\underline{I}_0}$$

will be studied, where $\underline{U}_2$ is the output voltage and $\underline{I}_0$ is the source current in the circuit given below.



1. Consider first extreme values of the radian frequency $\omega = 0$ and $\omega = \infty$. Give the expressions for the transfer impedance in these cases. Plot the equivalent circuits at $\omega = 0$ and $\omega = \infty$.

	$\omega = 0$	$\omega \to \infty$
Equivalent circuit		
$\underline{Z}_T(\omega) = \frac{\underline{U}_2}{\underline{I}_0}$	$R_1 + R_2$	0

2. Give the expression for the complex transfer impedance as a function of the radian frequency ω assuming $R_1 = R_2 = R$ and $C_1 = C_2 = C$.

$$\underline{Z}_T = \frac{2 \cdot R + j\omega CR^2}{1 - (\omega CR)^2 + j3\omega CR}$$

3. Introduce the normalized frequency $\Omega = \omega \cdot R \cdot C$ and give $\underline{Z}_T(\Omega)$.

$$\underline{Z}_T = \frac{2 \cdot R + j\Omega R}{1 - \Omega^2 + j3\Omega}$$

4. Give the values of $\underline{Z}_T(\Omega)$ for $\Omega \to 0$ and $\Omega \to \infty$.

$$\underline{Z}_T(\Omega = 0) = 2 \cdot R \quad \underline{Z}_T(\Omega \to \infty) = 0$$

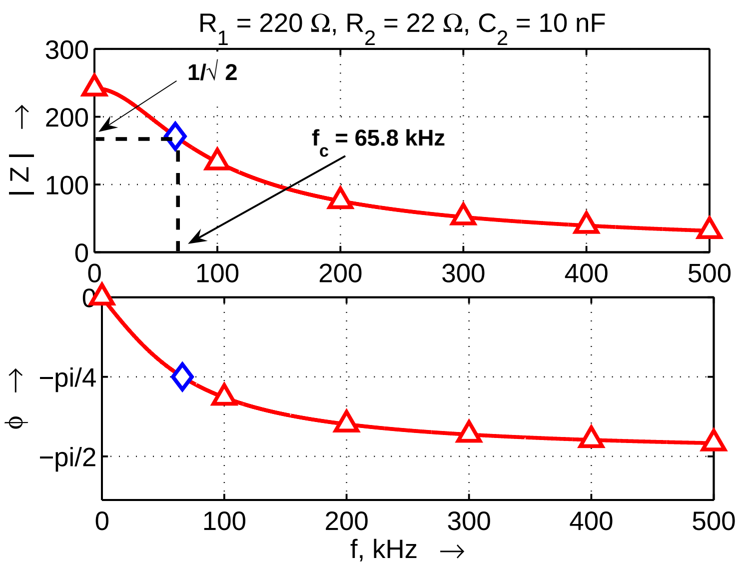
5. Determine the transfer impedance $\underline{Z}_T(\Omega)$ neglecting the capacitance C_1. Assume that $C_2 = C$ and $R_1 \neq R_2$.

$$\underline{Z}_T = \frac{R_1 + R_2}{1 + j\omega C \cdot (R_1 + R_2)}$$

6. Calculate the 3-dB-corner frequency for circuit configuration of the art before $R_1 = 220\ \Omega$, $R_2 = 22\ \Omega$ and $C = 10$ nF.

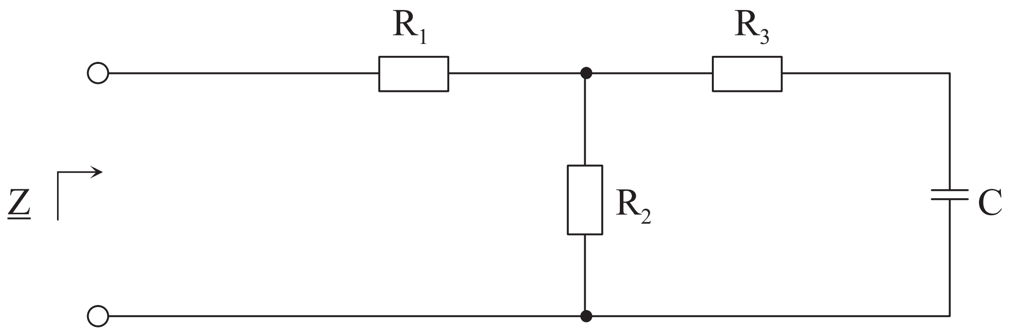
$$f_C = \frac{1}{2\pi C \cdot (R_1 + R_2)} = 65.8 \text{ kHz}$$

7. Plot the magnitude and the phase of the transfer impedance. Denote on the plot all characteristic points.



Problem 6: Locus diagram

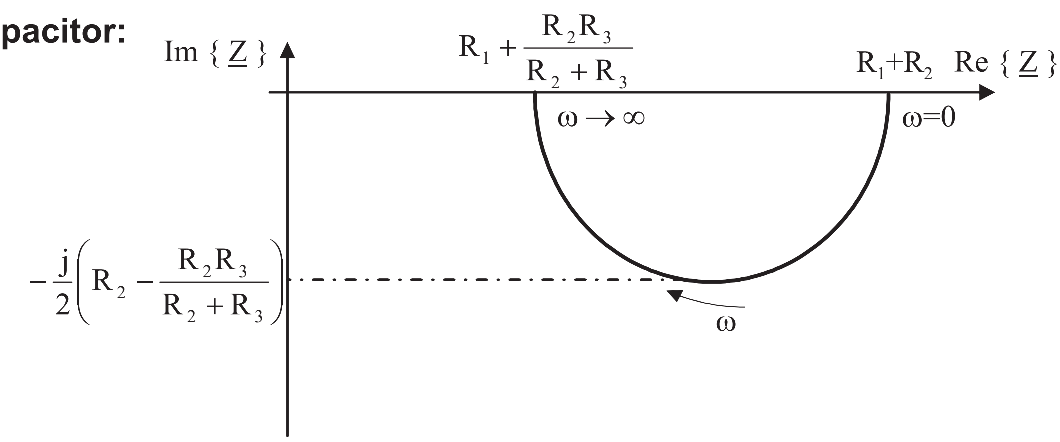
Find the locus diagram for the following circuit:



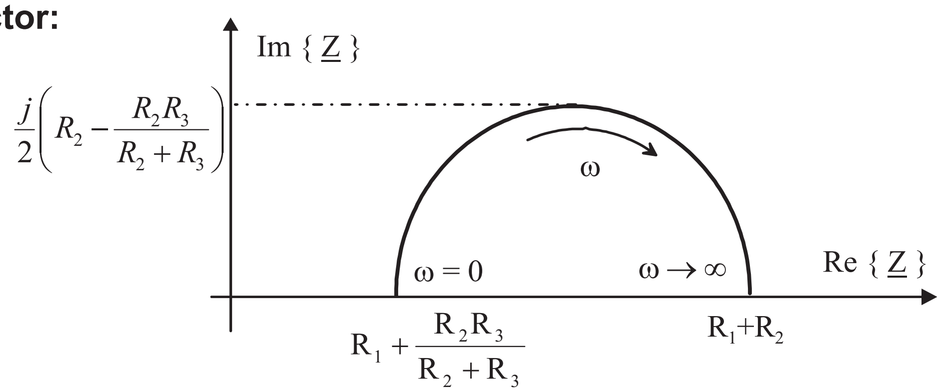
1. Plot stepwise the locus diagram of the circuit with frequency as a parameter.
2. Verify the curve by calculating the impedance directly from the circuit using the frequencies $\omega \to 0$ and $\omega \to \infty$.

Answer:

with capacitor:



with inductor:



Problem 7: Resonance

The circuit comprises an ideal sinusoidal current source, a series resistance $R_s = 100\ \Omega$ and a parallel resonance circuit with $L = 30\ \mu H$, $C = 0.5\ \mu F$, $R = 10\ \Omega$.

1. Find the resonance frequency, bandwidth and quality factor.

$$\omega_0 = 258.2 \cdot 10^3\ \frac{rad}{s};\ B = 220 \cdot 10^3\ \frac{rad}{s}\ ;\ Q = 1.17$$

2. Estimate the values of the half-power frequencies.

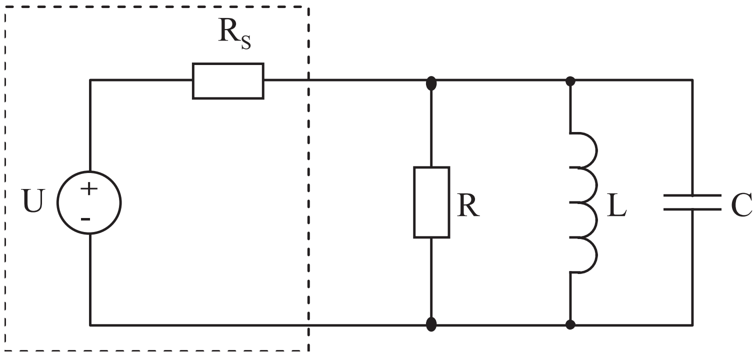
$$\omega_- = 170.6 \cdot 10^3\ \frac{rad}{s};\ \omega_+ = 390.6 \cdot 10^3\ \frac{rad}{s}$$

3. What is the influence of a source resistance?

A source resistance decreases the quality of the resonance circuit.
The resonance frequency remains the same $\omega_0 = 258.2 \cdot 10^3$

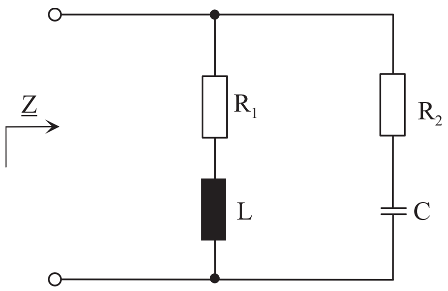
Without source resistance: $B = 200 \cdot 10^3\ \frac{rad}{s}\ ;\ Q = 1.29;$

$$\omega_- = 176.0 \cdot 10^3\ \frac{rad}{s};\ \omega_+ = 376.9 \cdot 10^3\ \frac{rad}{s}$$



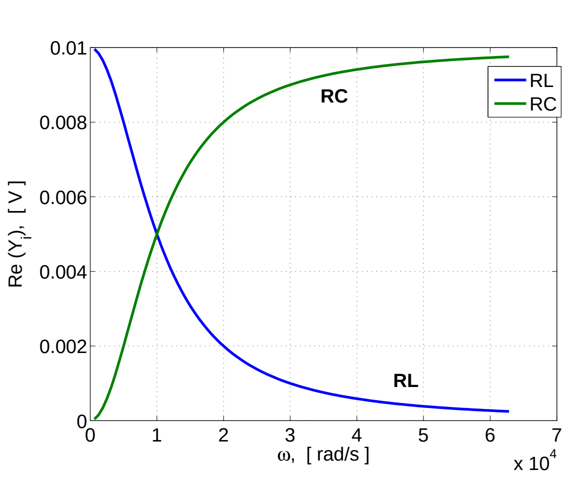
Problem 8: Circuit design

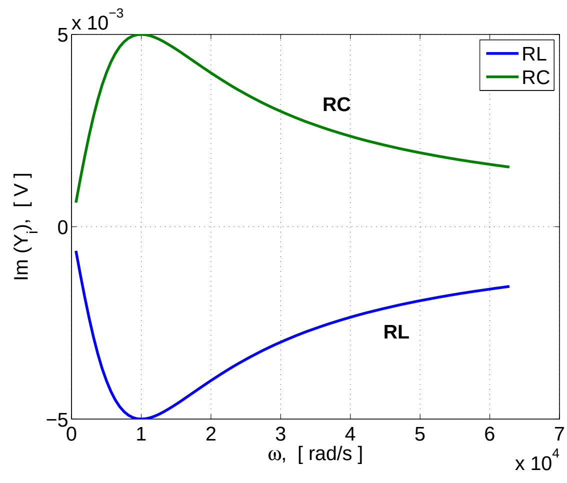
Find R_1, R_2 and C such that the input impedance $\underline{Z}$ for all frequencies equals 100 Ω. The inductance L is given $L = 10$ mH.



$R_1 = 100\,\Omega$
$R_2 = 100\,\Omega$
$C = 1\,\mu\text{F}$

Corner case 1:	$\omega = 0 => X_L$ - short circuit X_C - open circuit => $\underline{Z} = R_1 = 100\,\Omega$
Corner case 2:	$\omega \rightarrow \infty => X_L$ - open circuit X_C - short circuit => $\underline{Z} = R_2 = 100\,\Omega$
Some freq $\omega = \omega_0$:	$=> X_L = -X_C => j\omega_0 L = -\frac{1}{j\omega_0 C}$ $100\,\Omega = \omega_0 L = \frac{1}{\omega_0 C} => \omega_0 = \frac{X_L}{L}$ $C = \frac{1}{\omega_0 X_C} = \frac{L}{X_L^2} => C = 1\,\mu\text{F}$





Problem 9: Two-port (1)

For the circuit shown in the figure below with $R_p = 3\ k\Omega$, $g_m = 51$ mS, $C_1 = 50$ fF, $C_2 = 3.2$ fF and $C_3 = 34$ fF at f = 0.5 GHz

1. calculate the Y-parameters;

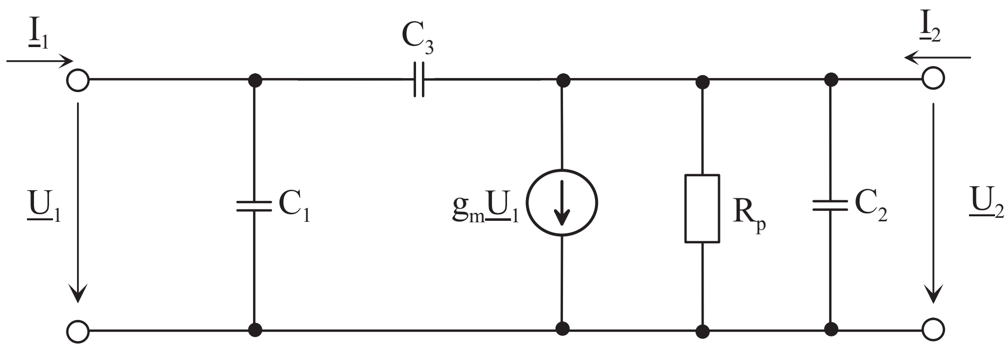
$$\left[\underline{Y}\right] = \begin{bmatrix} j\omega \cdot (C_1 + C_3) & -j\omega C_3 \\ g_m - j\omega C_3 & \frac{1}{R_P} + j\omega\left(C_2 + C_3\right) \end{bmatrix}$$

2. compare the voltage gain $\underline{U}_2/\underline{U}_1$ for $R_L = 50\ \Omega$ and $R_L = \infty$.

$$\underline{G}_V = \frac{\underline{U}_2}{\underline{U}_1} = -\frac{g_m - j\omega C_3}{\frac{1}{R_P} + j\omega\left(C_2 + C_3\right) + \frac{1}{R_L}}$$

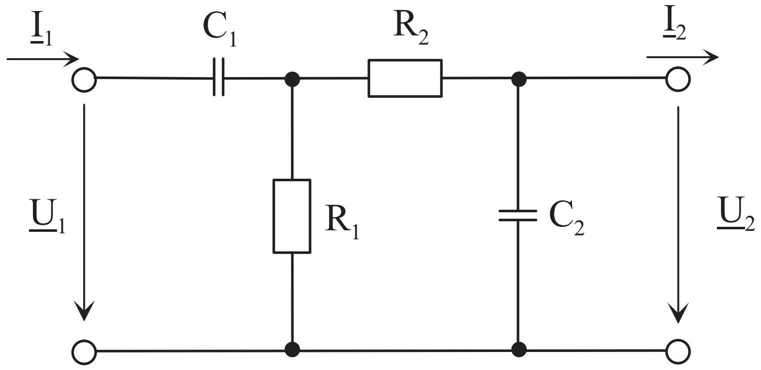
$$\left|\underline{G}_V\left(R_L = 50\Omega\right)\right| = 2.5 \hat{=} 8\text{ dB}$$

$$\left|\underline{G}_V\left(R_L \rightarrow \infty\right)\right| = 153 \hat{=} 44\text{ dB}$$



Problem 10: Two-port (2)

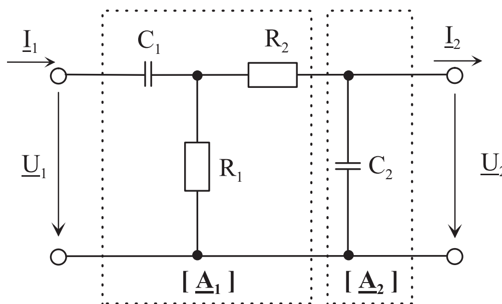
Determine the chain parameters of the two-port shown in the figure below:



1. using chain parameter definition;

$$[\underline{A}] = \begin{bmatrix} 1 + j\omega C_2 \cdot R_2 + \dfrac{1 + j\omega C_2\,(R_1 + R_2)}{j\omega C_1 R_1} & R_2 + \dfrac{1}{j\omega C_1} \cdot \left(1 + \dfrac{R_2}{R_1}\right) \\ \dfrac{1}{R_1} + j\omega C_2 \cdot \left(1 + \dfrac{R_2}{R_1}\right) & 1 + \dfrac{R_2}{R_1} \end{bmatrix}$$

2. using two sub-circuits in cascade connection.



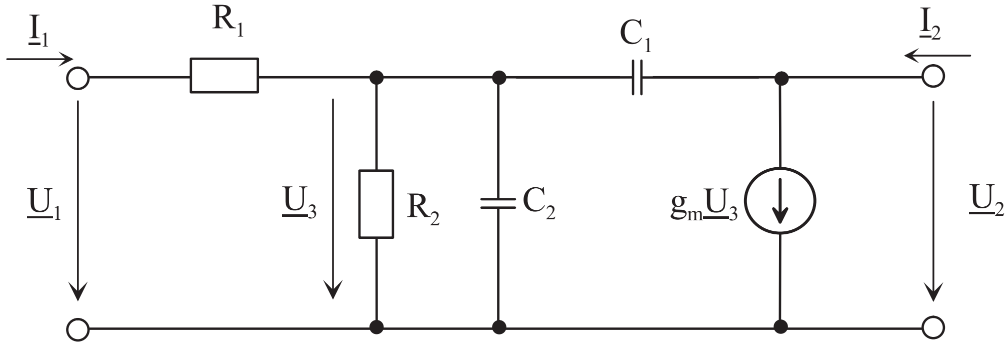
$$[\underline{A}_1] = \begin{bmatrix} 1 + \dfrac{1}{j\omega C_1 R_1} & R_2 + \dfrac{1}{j\omega C_1} + \dfrac{R_2}{j\omega C_1 R_1} \\ \dfrac{1}{R_1} & 1 + \dfrac{R_2}{R_1} \end{bmatrix}$$

$$[\underline{A}_2] = \begin{bmatrix} 1 & 0 \\ j\omega C_2 & 1 \end{bmatrix}$$

$$[\underline{A}] = [\underline{A}_1] \cdot [\underline{A}_1]$$

Problem 11: Two-port (3)

A small-signal equivalent circuit of a bipolar transistor in common-emitter configuration is shown in figure below. Determine the h-parameters of the given two-port.



$$[\underline{H}] = \begin{bmatrix} R_1 + \dfrac{1}{\dfrac{1}{R_2} + j\omega\left(C_1 + C_2\right)} & \dfrac{j\omega C_1}{\dfrac{1}{R_2} + j\omega C_1 + j\omega C_2} \\ \dfrac{g_m - j\omega C_1}{\dfrac{1}{R_2} + j\omega\left(C_1 + C_2\right)} & j\omega C_1 \cdot \dfrac{g_m - j\omega C_1}{\dfrac{1}{R_2} + j\omega\left(C_1 + C_2\right)} + j\omega C_1 \end{bmatrix}$$

Problem 12: Transistor models

1. An NMOS transistor has the following parameters:

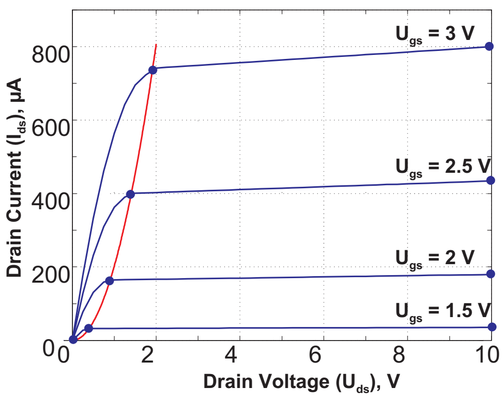
$W = 100\ \mu m$ $\quad L = 10\ \mu m$
$\lambda = 0.01 V^{-1}$ $\quad \mu_n = 1400\ cm^2/(V\cdot s)$
$t_{ox} = 0.12\ \mu m$ $\quad \phi_f = 0.3\ V$
$N_A = 10^{15}\ atom/cm^3$ $\quad \epsilon_{Si} = 11.7$
$\epsilon_{ox} = 3.9$ $\quad \epsilon_0 = 8.854\cdot 10^{-15} F/m$
$U_{to} = 1.1\ V$

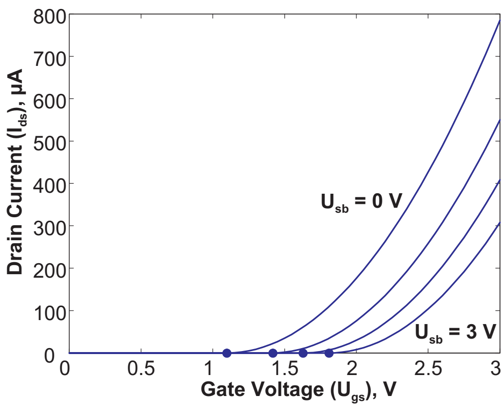
a) Sketch the Ids–Uds characteristics for U_{ds} from 0 to 10 V and U_{gs} =1.5, 2, 2.5 and 3 V. Assume $U_{sb} = 0$ V.

U_{gs}, V	1.5	2	2.5	3
U_{ds}, V	0.4	0.9	1.4	1.9
I_{ds}, μA	32.23	163.16	394.81	727.17
U_{gs}, V	1.5	2	2.5	3
U_{ds}, V	10	10	10	10
I_{ds}, μA	35.45	179.48	434.29	799.89

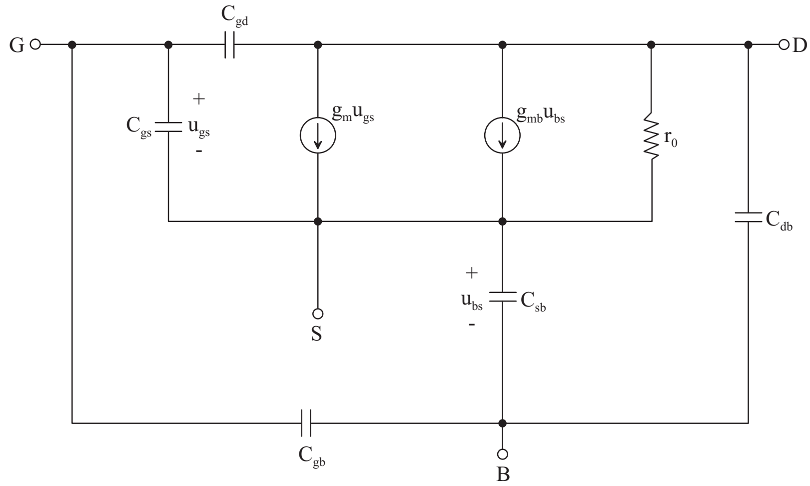
b) Sketch the Ids–Ugs characteristics for $U_{ds} = 8$ V as U_{gs} varies from 0 to 3 V with $U_{sb} = 0, 1, 2$ and 3 V.

U_{sb}, V	0	1	2	3
U_{th}, V	1.1	1.41	1.63	1.81





2. Derive and sketch the complete small-signal equivalent circuit for the device of Problem 1 with $U_{gs} = 2$ V, $U_{ds} = 10$ V, and $U_{sb} = 1$ V. Use $\psi_0 = 0.6$ V, $C_{sb0} = C_{db0} = 0.2$ pF $C_{gb} = 0.08$ pF. Overlap capacitance from gate to source and gate to drain is 0.02 pF.



$g_m = 0.26$ mS $C_{db} = 45.4$ fF
$g_{mb} = 66\ \mu$S $C_{gs} = 211$ fF
$r_0 = 1.3$ MΩ $C_{gd} = 20$ fF
$C_{sb} = 122$ fF $C_{gb} = 80$ fF

3. Use the device data of Problems 1 and 2 to calculate the frequency of unity current gain of the transistor with $U_{ds} = 5$ V, $U_{sb} = 0$ V, and $U_{gs} = 1.5$, 2 and 3 V.

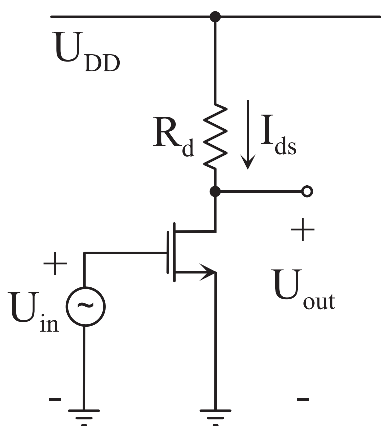
U_{gs}, V	1.5	2	3
f_t, MHz	86	194	410

Problem 13: Basic Single Transistor Amplifier Stage

For the common-source amplifier of the figure below calculate the small-signal voltage gain and the bias values of U_{in} and U_{out} at the edge of the triode region. Also calculate the bias values of U_{in} and U_{out} where the small-signal voltage gain is unity.

Assume:

$U_{DD} = 5$ V	$\lambda = 0$ V^{-1}
$\mu_n \cdot C'_{ox} = 40\ \mu$A/V^2	$U_{th} = 0.8$ V
W=100 μm	$R_D = 5$ kΩ
L=10 μm	



1. Calculate the small-signal voltage gain and the bias values of U_{in} and U_{out} at the edge of the triode region.

$$U_{ds} = 1.79 \text{ V}$$
$$U_{gs} = 2.6 \text{ V}$$
$$G_V = -3.58 \text{ V}$$

2. Calculate the bias values of U_{in} and U_{out} where the small-signal voltage gain is unity.

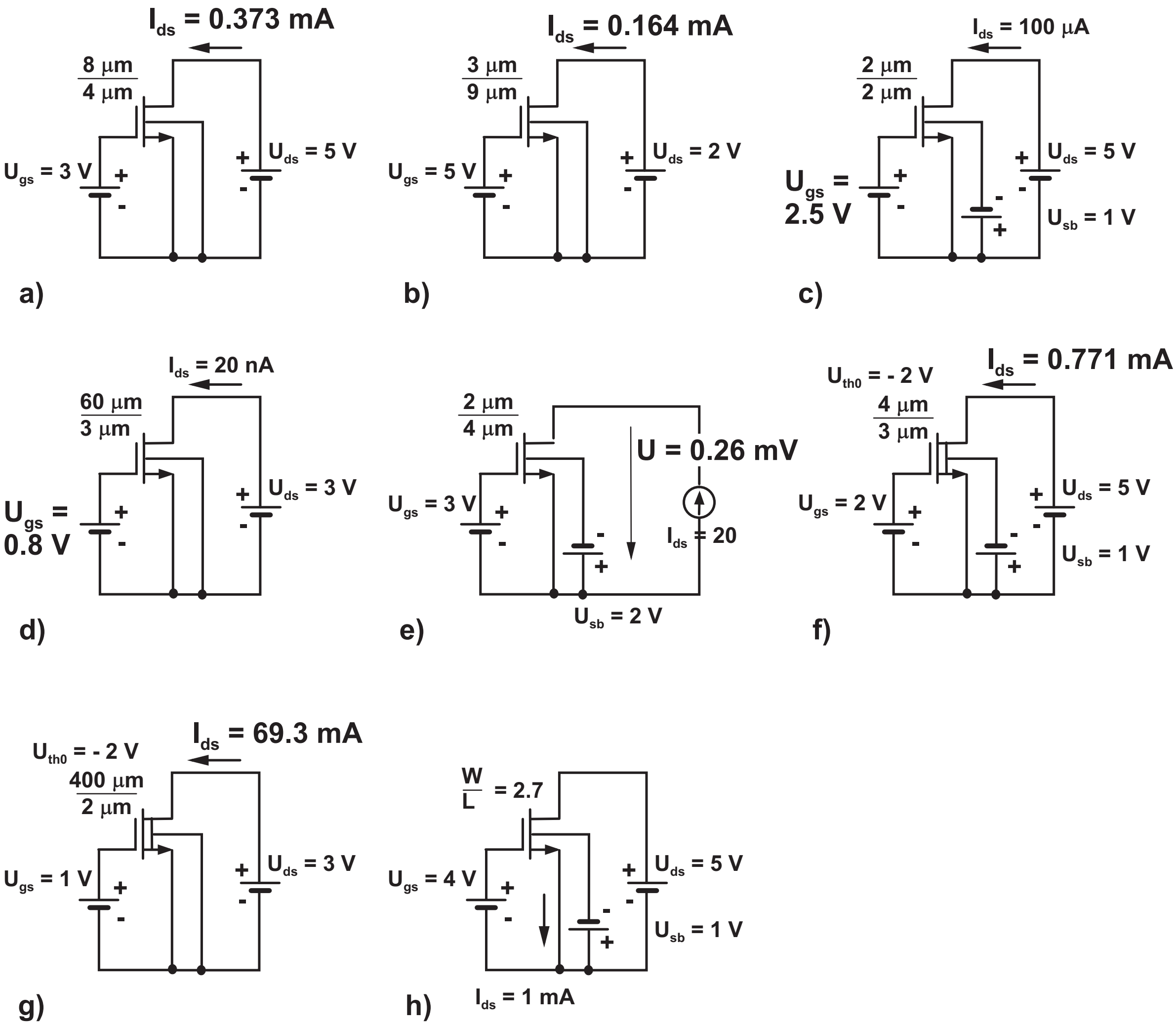
$$U_{ds} = 4.75 \text{ V}$$
$$U_{gs} = 1.3 \text{ V}$$

Problem 14: nMOS Transistors

Calculate the quantities indicated in the figure below, assuming the following parameters:

$\mu_\mathrm{n} \cdot \mathrm{C_{ox}} = 77\ \mu\mathrm{A/V^2}$, $U_{th,\,0} = 0.8$ V, $\phi_f = 0.76$ V, $\gamma = 0.35\ \mathrm{V}^{1/2}$ and $\lambda = 0\ \mathrm{V}^{-1}$

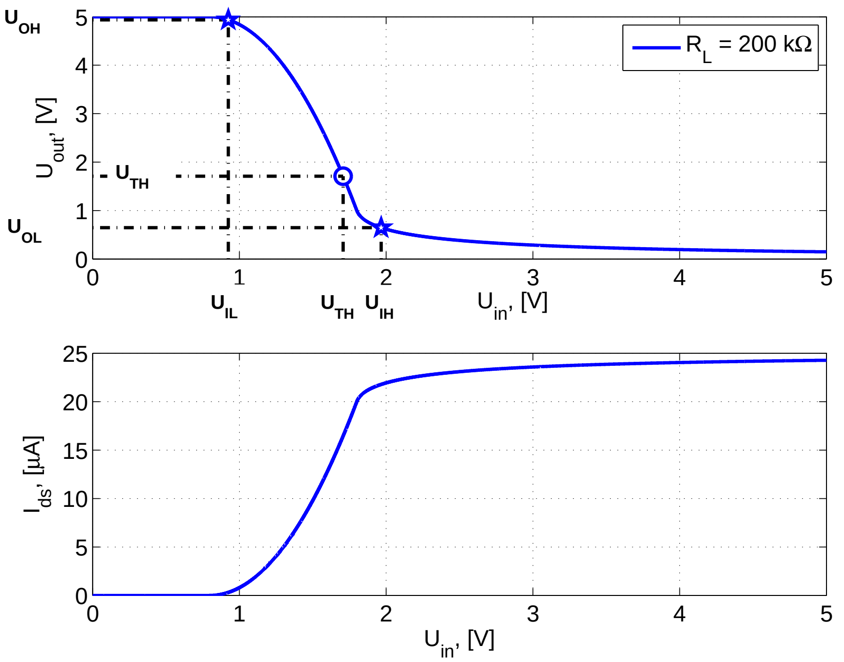
The quantities next to the devices in a form a/b mean $W = a$ and $L_g = b$.



Problem 15: Resistive-load inverter

Consider a resistive-load inverter circuit with $k_n = 40\ \mu\text{A/V}^2$, $U_{th,0} = 0.8$ V, $R_L = 200$ kΩ and $U_{DD} = 5$ V.

1. Sketch the voltage transfer characteristics of the inverter and calculate the critical points (U_{OL}, U_{OH}, U_{IL} and U_{IH}).



$U_{OL} = 0.147$ V
$U_{IL} = 0.925$ V
$U_{IH} = 1.97$ V
$U_{OH} = 5$ V
$U_{TH} = 1.7$ V

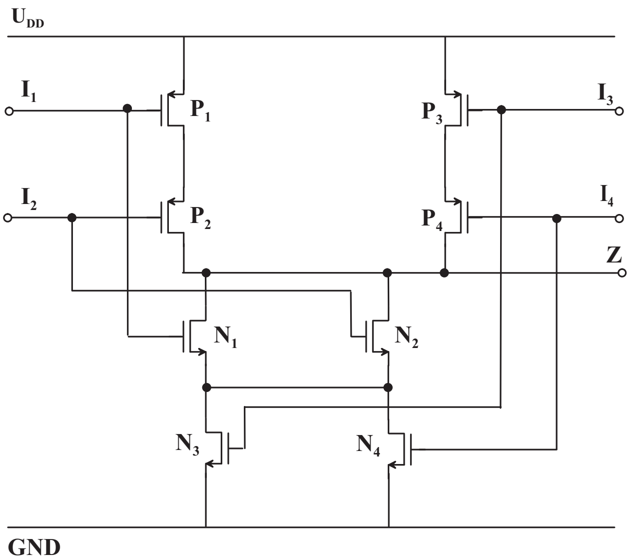
2. Find the noise margins of the circuit. Comment the quality of the inverter.

$NM_L = 0.28$ V
$NM_H = 2.97$ V
NM_L is too low.
It should be about 25% of U_{DD}.

Problem 16: Digital CMOS Circuits

Create truth tables and transistor-level schematics using CMOS logic gates for the following functions:

1. $\overline{\mathrm{Q}} = [\mathrm{I}_1 + \mathrm{I}_2] \cdot [\mathrm{I}_3 + \mathrm{I}_4]$



2. $\mathrm{Z} = \left(\bar{\mathrm{I}}_1 \cdot \bar{\mathrm{I}}_2\right) + \bar{\mathrm{I}}_3 + \bar{\mathrm{I}}_4$